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STUDIES OF UNSTEADY VISCOUS FLOWS USING A  
TWO-EQUATION MODEL OF TURBULENCE

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## INTRODUCTION

The objective of this research effort is to develop a two equation model of turbulence, based on the turbulent kinetic energy and energy dissipation, suitable for prediction of unsteady viscous flows. A second objective of this research is to compare the performance of the two equation model with simpler algebraic models such as the Baldwin-Lomax two layer eddy viscosity model, and a model by Johnson and King which accounts for upstream history of the turbulent kinetic energy.

## SUMMARY OF PROGRESS

The NASA Ames version of the Johnson and King model subroutine was acquired, and was modified for use in the Georgia Tech 2-D Navier-Stokes code, and the NASA Lewis version of the ARC2D code. The usefulness of the Johnson and King model was evaluated by performing a number of calculations with the ARC2D-Johnson-King solver. In Figure 1, the surface pressure distribution and displacement thickness variation over a RAE 2822 airfoil at 0.75 Mach number, 3.19 degree angle of attack and 6.2 Million Reynolds number is shown. For comparison, a set of experimental data and results from the ARC2D-Baldwin-Lomax solver is also shown. It is seen that the Johnson and King model does a slightly better job of predicting the shock location, and the rapid rise in the displacement thickness downstream of the shock. In Figure 2, the velocity profiles on the upper surface at two chordwise locations are shown. Again, the Johnson and King model performs somewhat better than the Baldwin-Lomax model.

Calculations have also been carried out for an iced airfoil configuration at 0.12 Mach number, 1.141 Million Reynolds number for a range of angles of attack. In Figure 3, the surface pressure distribution at zero angle of attack as predicted by the two turbulence models (Baldwin-Lomax and Johnson-King) are shown. In Figure 4, the turbulent eddy viscosity contours and streamline contours for the above condition are also shown. It appears that the Baldwin-Lomax model tends to overpredict the turbulent eddy viscosity everywhere, which results in a slightly shorter separation bubble. Turbulent eddy viscosity contours for the same airfoil at 4 degree angle of attack, shown in Figure 5, show a similar behavior.

The Johnson-King model requires more computer time to obtain a final solution than the Baldwin-Lomax model. Further, some experience with the Johnson-King model is needed to know at what location over the airfoil the Johnson-King ODE model should be turned on. Work is in progress to gain additional experience with the Johnson-King model.

The two equation turbulence model ( $k-\epsilon$ ) has been coded, and incorporated into the Georgia Tech 2-D Navier-Stokes solver. Results for the iced airfoil configuration using the  $k-\epsilon$  model will be presented in the next progress report.

Appendix I gives a brief description of the equations used in the three turbulence models under study.

## APPENDIX I

### TURBULENCE MODEL (I)

#### BALDWIN-LOMAX ALGEBRAIC EDDY VISCOSITY MODEL

$$\mu_t = \begin{cases} \mu_t \text{ inner} & y \leq y_{\text{crossover}} \\ \mu_t \text{ outer} & y > y_{\text{crossover}} \end{cases}$$

$$\mu_t \text{ inner} = \rho l^2 |\omega|$$

$$l = 0.4 y [ 1 - \exp(-y^+/A^+) ], \quad y^+ = \rho u_{\tau} y / \mu_w$$

$$\mu_t \text{ outer} = 0.0168 \times 1.6 \rho F_{\text{wake}} F_{\text{kleb}}$$

$$F_{\text{wake}} = \min \{ y_{\text{max}} F_{\text{max}}, 0.25 y_{\text{max}} U^2_{\text{dif}} / F_{\text{max}} \}$$

$$F(y) = y |\omega| [ 1 - \exp(-y^+/A^+) ]$$

$$F = \text{velocity scale}; \quad \omega = \text{length scale}$$

$$F_{\text{kleb}} = 1 / [ 1 + 5.5 (0.3 y / y_{\text{max}})^6 ]$$

- Turbulence velocity scale and turbulence length scale are determined by the algebraic relations.
- Equilibrium assumption: turbulent shear stress depends only on local properties of the mean flow.
- Work well for attached and slightly separated flows.  
Some ambiguity to determine  $F_{\text{max}}$  for strong separated flows
- Easy to apply; least computational effort.

## TURBULENCE MODEL (II)

### JOHNSON-KING ODE MODEL

- An assumed eddy viscosity distribution

$$\mu_t = \mu_{to} [ 1 - \exp(-\mu_{ti}/\mu_{to}) ]$$

$$\mu_{t \text{ inner}} = D^{2/3} 0.4 y (-\overline{u'v'})_m^{1/2}$$

$$\mu_{t \text{ outer}} = (x) 0.0168 u_e \delta^* F_{kleb}$$

$$D = 1 - \exp[ -(-\overline{u'v'})_m^{1/2} \rho_w y / \mu_w A^+ ]$$

$$(-\overline{u'v'})_m^{1/2} = \text{velocity scale}, \quad \delta^* = \text{length scale}$$

- An ODE for the maximum Reynolds shear stress

$$\frac{dg}{dx} = \frac{a_1}{2u_m L_m} \left[ \left(1 - \frac{g}{g_{eq}}\right) + \frac{C_{dif} L_m}{a_1 \delta [0.7 - (y/\delta)_m]} \left| 1 - \left(\frac{\mu_{to}}{\mu_{to,eq}}\right)^{1/2} \right| \right]$$

$$g = (-\overline{u'v'})_m^{-1/2}; \quad g_{eq} = (-\overline{u'v'})_{m,eq}^{-1/2}$$

- Employ an ODE relating to the turbulence velocity scale.
- Non-equilibrium model: turbulent shear stress is influenced by "history" effects of streamwise pressure gradient.
- Performs better for separated flow with severe adverse pressure gradient than the algebraic models.
- Not so robust as the algebraic models; needs more computational effort than the algebraic models.

### TURBULENCE MODEL (III)

#### GORSKI'S k-ε TWO EQUATIONS MODEL

$$k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2}) \quad \epsilon = \nu [(\partial u' / \partial y)^2 + (\partial v' / \partial x)^2]$$

kinetic energy                      dissipation rate

- Outside the viscous sublayer

$$\partial_t Q_1 + \partial_x E_1 + \partial_y F_1 = \partial_x R_1 + \partial_y T_1 + S_1$$

convection                      diffusion                      source  
term                                  term                                  term

$$Q_1 = \begin{pmatrix} \rho k \\ \rho \epsilon \end{pmatrix}, \quad E_1 = \begin{pmatrix} \rho k u \\ \rho \epsilon u \end{pmatrix}, \quad F_1 = \begin{pmatrix} \rho k v \\ \rho \epsilon v \end{pmatrix}, \quad R_1 = \begin{pmatrix} \mu_k \partial_x k \\ \mu_\epsilon \partial_x \epsilon \end{pmatrix}, \quad T_1 = \begin{pmatrix} \mu_k \partial_y k \\ \mu_\epsilon \partial_y \epsilon \end{pmatrix}$$

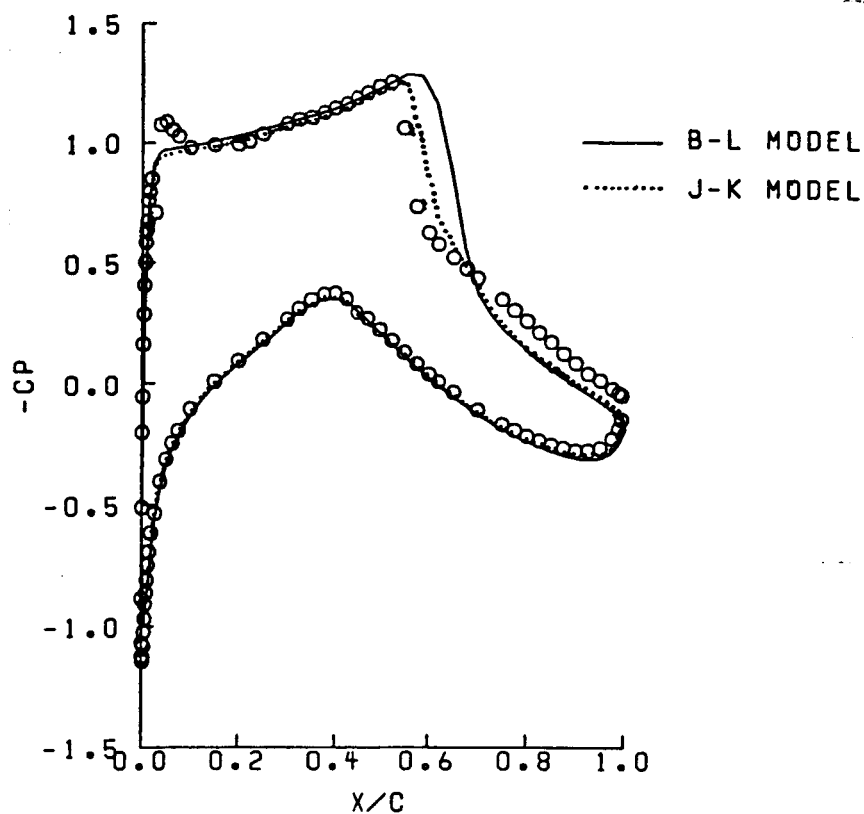
$$S_1 = \begin{pmatrix} P - \epsilon \\ C_1 P \epsilon / k - C_2 \epsilon^2 / k \end{pmatrix}$$

$$P = \text{production} = \mu_t [(\partial_y u)^2 + (\partial_x v)^2 + 2 \partial_y u \partial_x v]$$

$$\mu_t = \text{eddy viscosity} = C_\mu k^2 / \epsilon$$

- Within the viscous sublayer, k, ε and μ<sub>t</sub> are determined through the algebraic relations.

- Employs two PDE's for relating the turbulence length scale and turbulence velocity scale.
- Non-equilibrium model; employs more physics than the other two models.
- Generally it predicts better mean flow properties and turbulent shear stress for separated flows than the algebraic model and the one-equation model.
- Not easy to apply; requires most computational effort among the eddy viscosity models.



PRESSURE DISTRIBUTIONS FOR  
RAE 2822 AIRFOIL,  $M_{\infty}=0.750$ ,  
 $A=3.19$ ,  $RE=6.20$  MILLION.

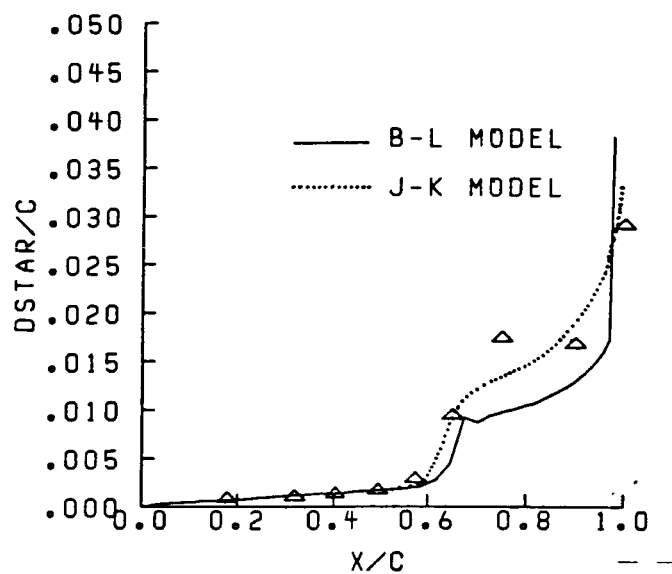


FIG. 1 UPPER SURFACE OF RAE 2822 AIRFOIL  
 $A=3.19$ ,  $M_{\infty}=0.750$ ,  $RE=6.20$  MILLION

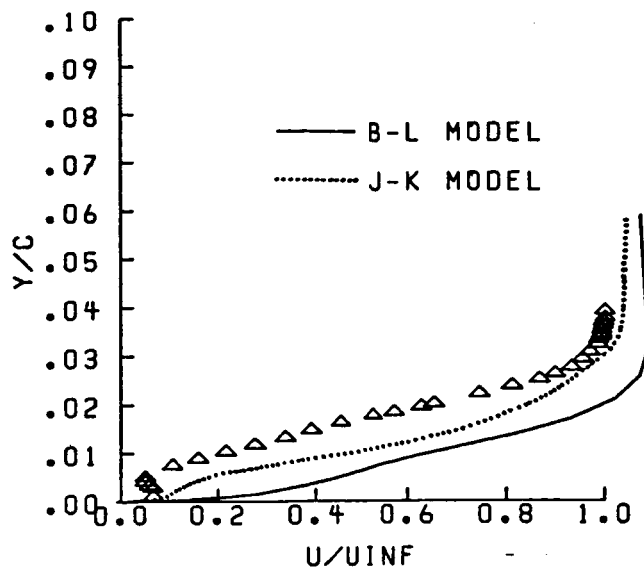


FIG. VELOCITY PROFILE AT  $X/C=0.750$ ,  
UPPER SURFACE OF RAE 2822 AIRFOIL,  
 $A=3.19$ ,  $M_{\infty}=0.750$ ,  $Re=6.20$  MILLION.

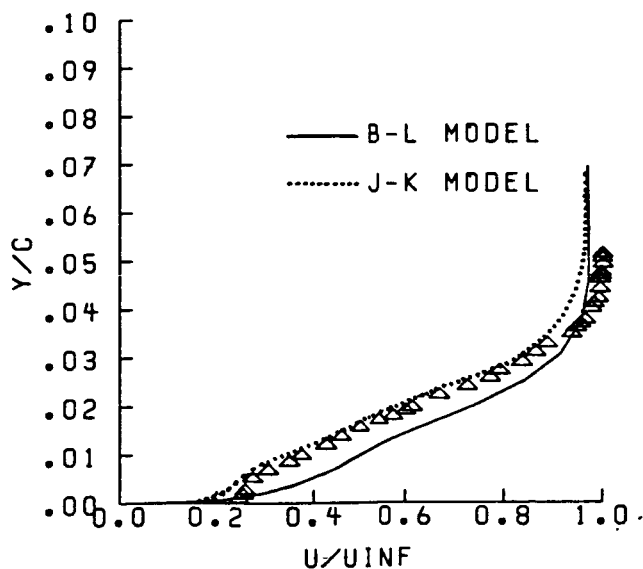
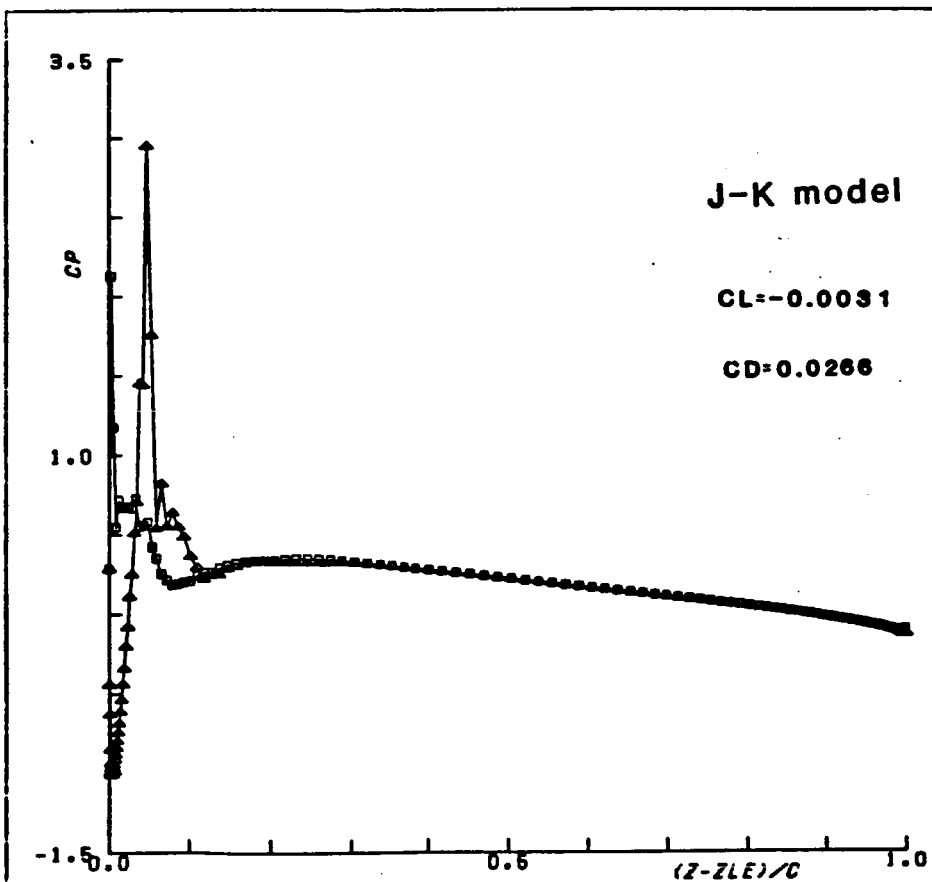
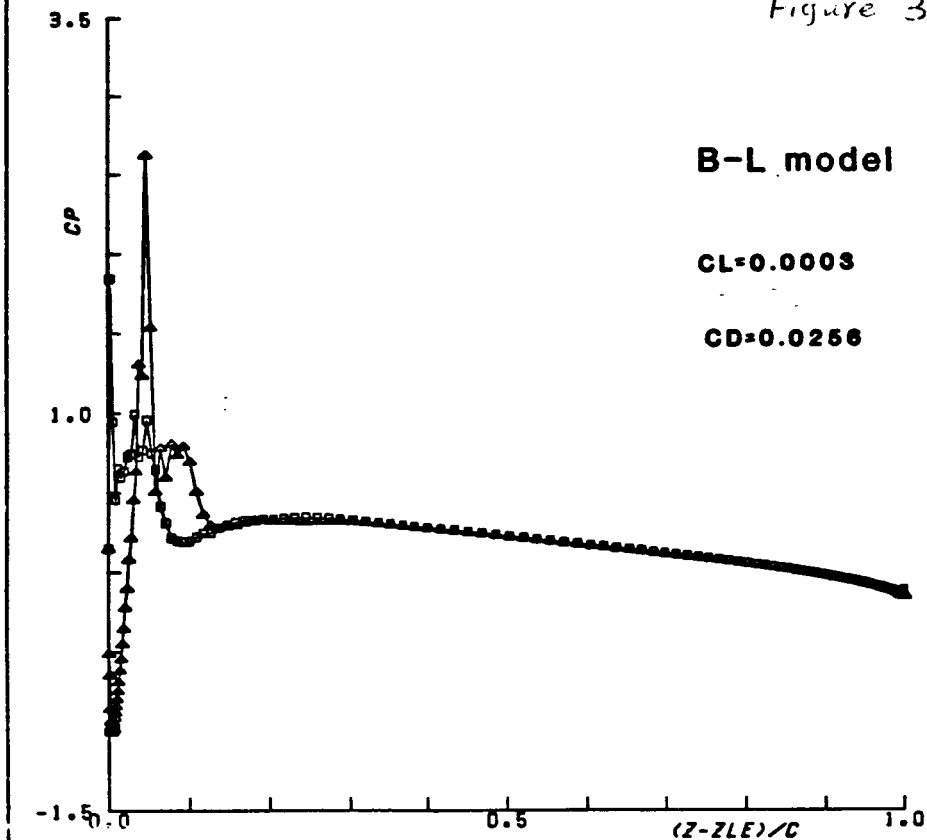


FIG. 2 VELOCITY PROFILE AT  $X/C=0.90$ ,  
UPPER SURFACE OF RAE 2822 AIRFOIL,  
 $A=3.19$ ,  $M_{\infty}=0.750$ ,  $Re=6.20$  MILLION



Figure 3



ARC2D ISOLATED AIRFOIL CODE

MACH 0.120 RE 11750000. ALPHA 0.00 ITER 10000

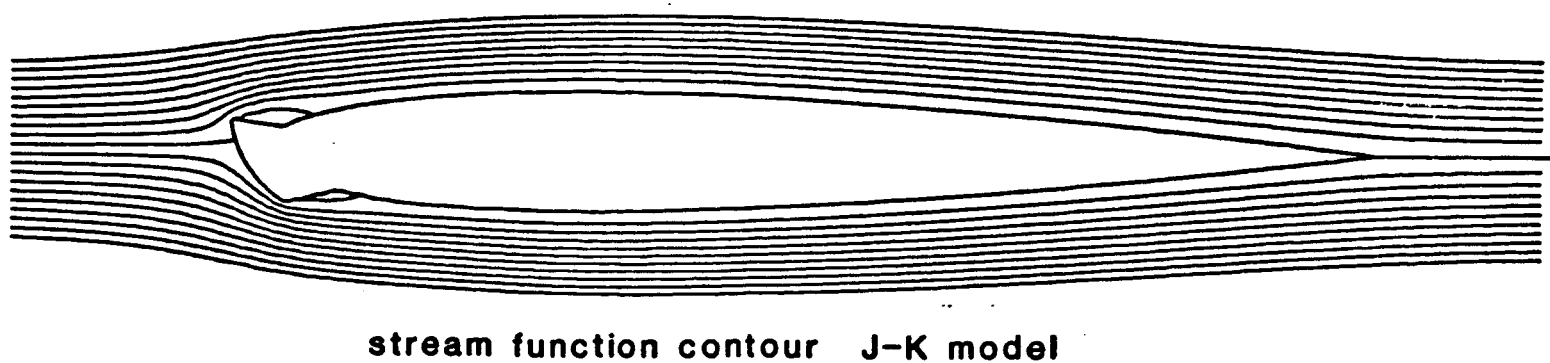
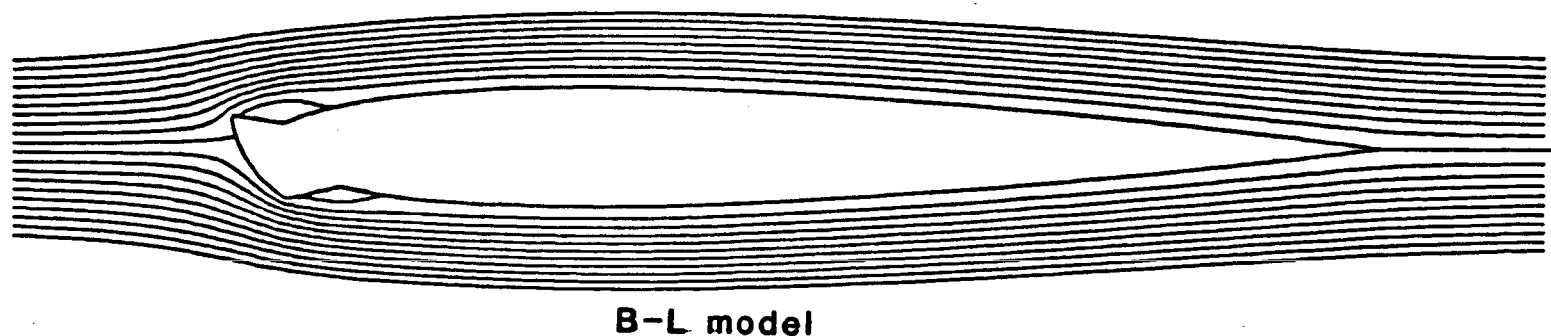
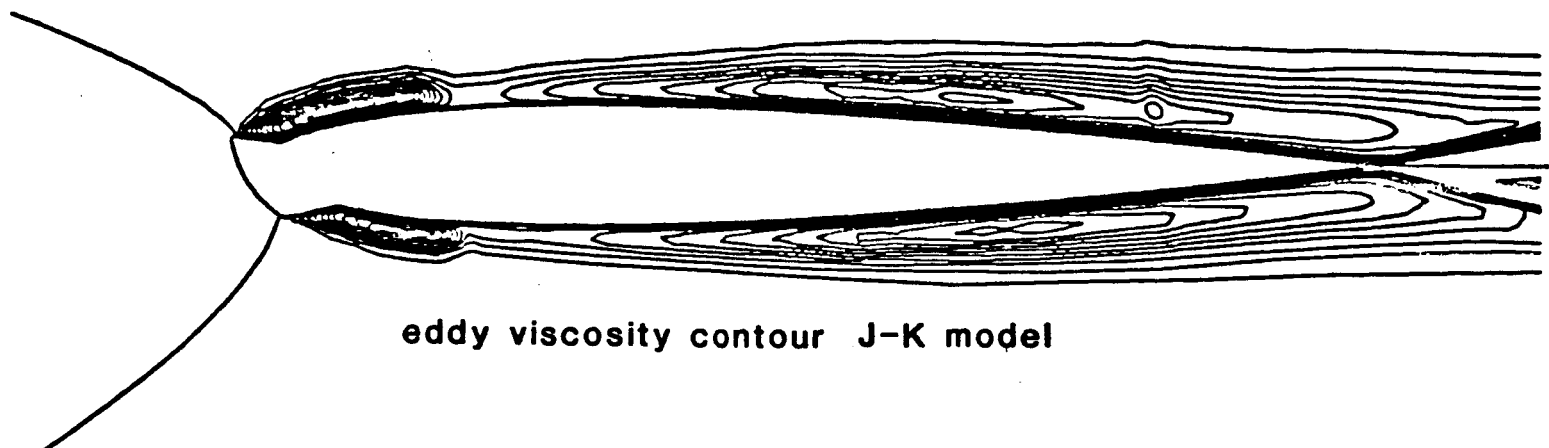
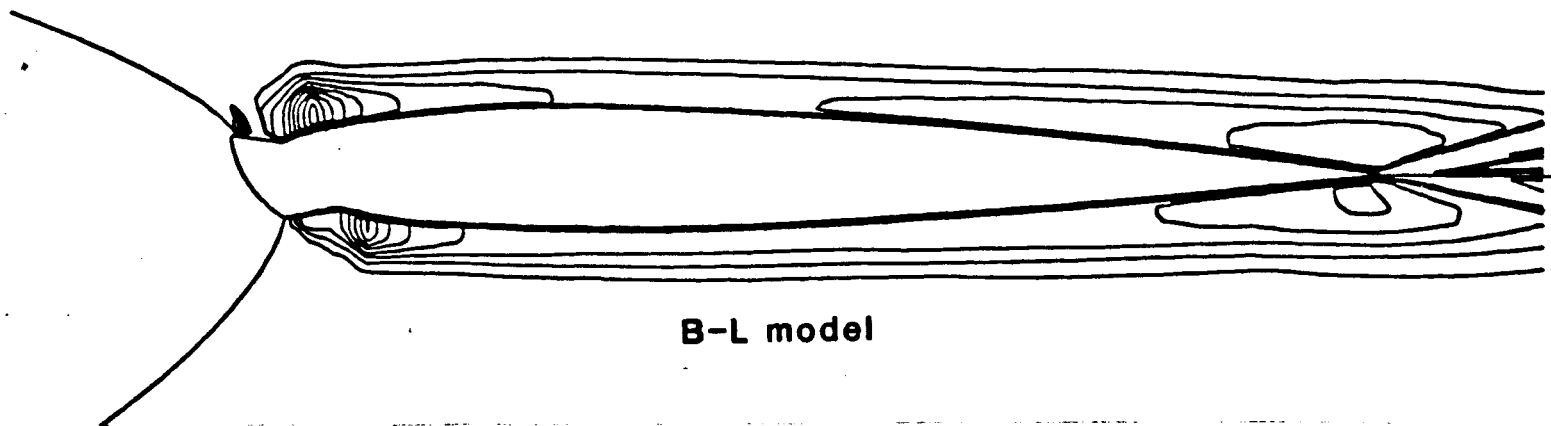
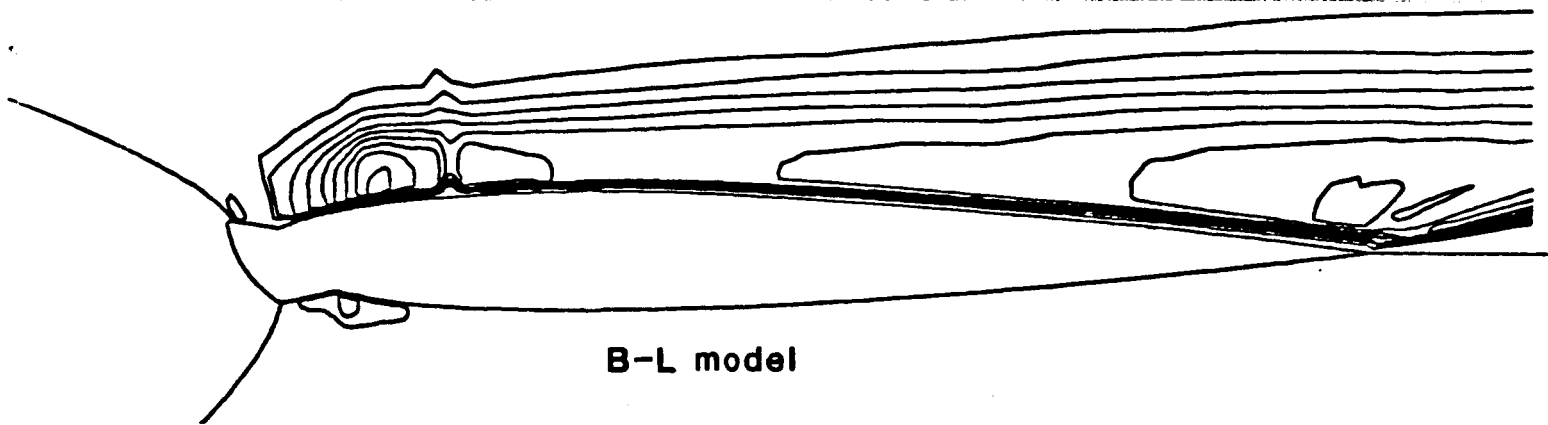
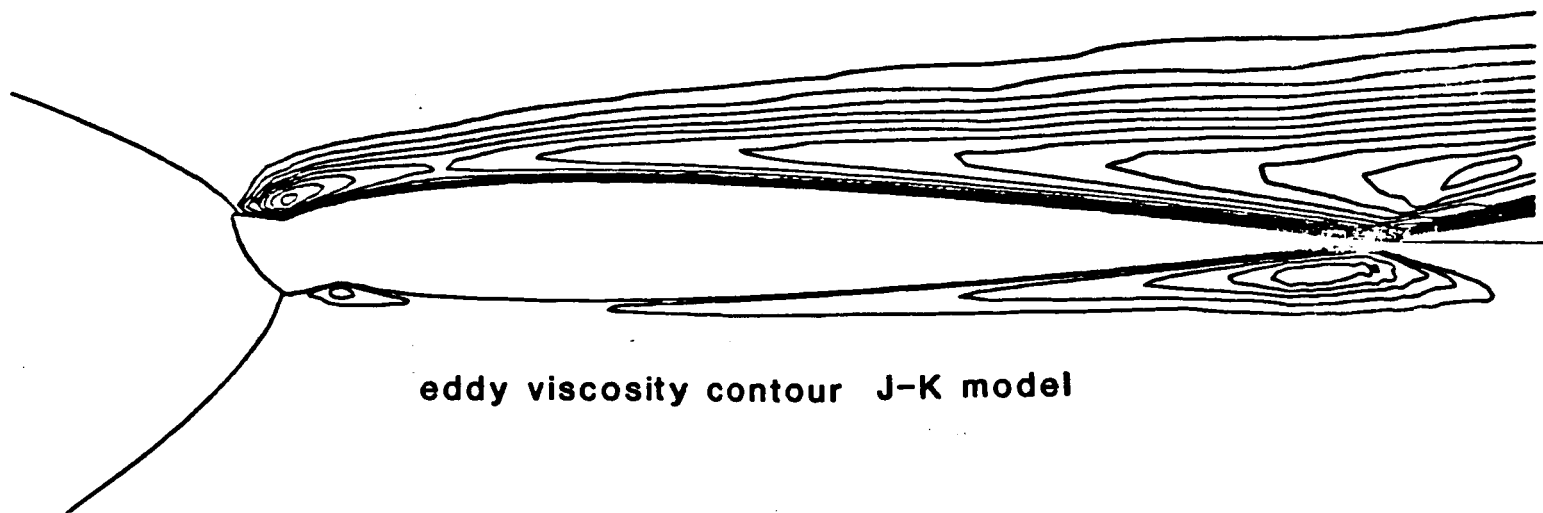


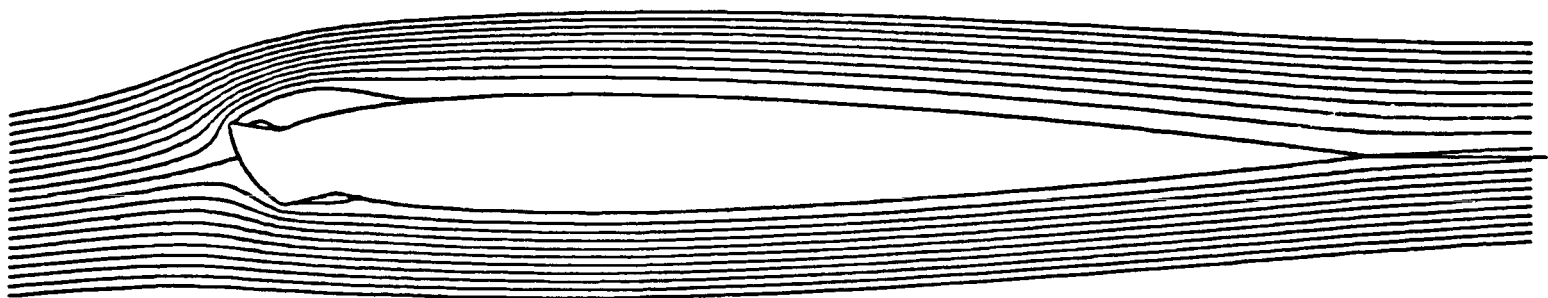
Figure 4  $M=0.12$   $\alpha=0^\circ$   $Re=1.41$  million



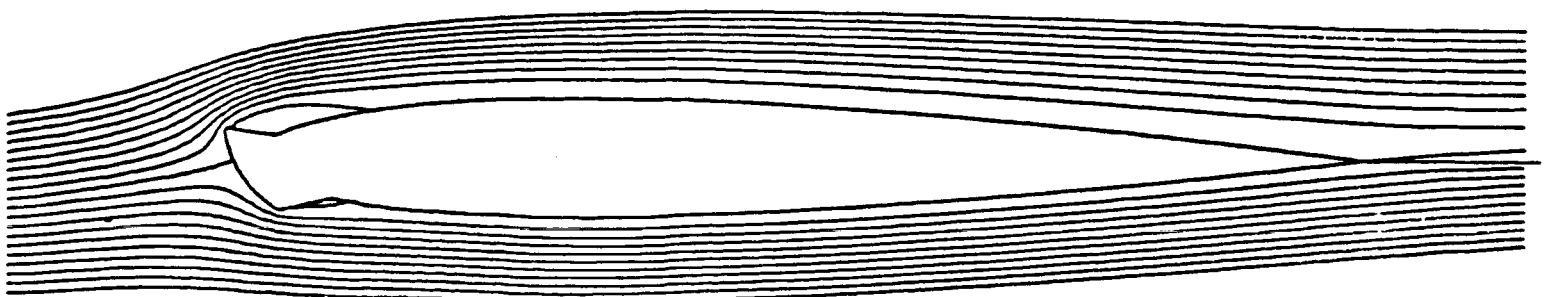
**B-L model**



**eddy viscosity contour J-K model**



**B-L model**



**stream function contour J-K model**

*Figure 5*  $M=0.12$   $\alpha=4^\circ$   $Re=1.41$  million